II. ZÁRTHELYI MEGOLDÁSOK

2003. december 15.

A.

1. $$\int \frac{\cos x}{2+\sin^2 x}\,dx = \frac{1}{\sqrt{2}}\cdot\int \frac{\frac{1}{\sqrt{2}}\cdot\cos x}{1+\left(\frac{1}{\sqrt{2}}\cdot\sin x\right)^2}\,dx = \frac{1}{\sqrt{2}}\cdot\operatorname{arctg}\left(\frac{1}{\sqrt{2}}\cdot\sin x\right)+c$$

2. $$\int \frac{x-1}{\sqrt{3+2x-x^2}}\,dx = -\int \frac{2-2x}{2\cdot\sqrt{3+2x-x^2}}\,dx = -\sqrt{3+2x-x^2}+c$$

3. $$\int (10x^4-\sin 2x)\cdot(2x^5+\cos^2 x+10)^6\,dx = \frac{1}{7}\cdot(2x^5+\cos^2 x+10)^7+c$$

4. $$\int 2^{2x+1}\cdot\sqrt{4^x+4}\,dx = \frac{2}{\ln 4}\cdot\int 4^x\cdot\ln 4\cdot\sqrt{4^x+4}\,dx = \frac{1}{\ln 2}\cdot\frac{2}{3}\cdot\sqrt{(4^x+4)^3}+c$$

5. $$\int \underbrace{(1-x)}_{u}\cdot\underbrace{\cos 4x}_{\substack{v' \\ v=\frac{1}{4}\cdot\sin 4x}}\,dx = (1-x)\cdot\left(\frac{1}{4}\cdot\sin 4x\right)-\int\left(-\frac{1}{4}\cdot\sin 4x\right)dx = (1-x)\cdot\left(\frac{1}{4}\cdot\sin 4x\right)-\frac{1}{16}\cdot\cos 4x+c$$

6. $$\int \sqrt{x}\cdot\operatorname{arctg}\sqrt{x^3}\,dx \underbrace{=}_{\substack{t=\sqrt{x^3} \\ dt=\frac{3}{2}\cdot\sqrt{x}\,dx}} \frac{2}{3}\cdot\int \underbrace{1}_{\substack{u' \\ u=t}}\cdot\underbrace{\operatorname{arctg} t}_{v}\,dt = \frac{2}{3}\cdot\left(t\cdot\operatorname{arctg} t-\int\frac{t}{1+t^2}\,dt\right)=$$

$$=\frac{2}{3}\cdot\left(t\cdot\operatorname{arctg} t-\frac{1}{2}\cdot\ln\left|1+t^2\right|\right)+c = \frac{2}{3}\cdot\sqrt{x^3}\cdot\operatorname{arctg}\sqrt{x^3}-\frac{1}{3}\cdot\ln(1+x^3)+c \qquad (x>0)$$

7. $$\int\frac{4x-1}{x^2-2x+10}\,dx = 2\cdot\int\frac{2x-2}{x^2-2x+10}\,dx+\int\frac{3}{(x-1)^2+9}\,dx=$$

$$=2\cdot\ln\left|x^2-2x+10\right|+\int\frac{\frac{1}{3}}{\left(\frac{x-1}{3}\right)^2+1}\,dx = 2\cdot\ln\left|x^2-2x+10\right|+\operatorname{arc\,tg}\left(\frac{x-1}{3}\right)+c$$

8. $$\frac{3x+4}{x^2+x-6}=\frac{A}{x-2}+\frac{B}{x+3}=\frac{A\cdot(x+3)+B\cdot(x-2)}{x^2+x-6} \Rightarrow \begin{matrix} A+B=3 \\ 3A-2B=4\end{matrix} \Rightarrow A=2,\ B=1, \quad \text{s így}$$

$$\int\frac{3x+4}{x^2+x-6}\,dx = \int\frac{2}{x-2}\,dx+\int\frac{1}{x+3}\,dx = 2\cdot\ln|x-2|+\ln|x+3|+c$$

9. $$\int\sin^3(x+1)\cdot\cos^4(x+1)\,dx = \int\sin(x+1)\cdot(1-\cos^2(x+1))\cdot\cos^4(x+1)\,dx = -\frac{\cos^5(x+1)}{5}+\frac{\cos^7(x+1)}{7}+c$$

10. $$\int\frac{4}{5+3\cos x}\,dx \underbrace{=}_{\substack{t=\operatorname{tg}\frac{x}{2},\ \cos x=\frac{1-t^2}{1+t^2} \\ dx=\frac{2}{1+t^2}\,dt}} \int\frac{4}{5+3\cdot\frac{1-t^2}{1+t^2}}\cdot\frac{2}{1+t^2}\,dt = \int\frac{1}{1+\left(\frac{1}{2}t\right)^2}\,dt = 2\cdot\operatorname{arc\,tg}\left(\frac{1}{2}\cdot\operatorname{tg}\frac{x}{2}\right)+c$$