II. ZÁRTHELYI MEGOLDÁSOK

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1. $$\int \frac{1}{(1+x)\cdot\sqrt{x}}\,dx = 2\cdot\int \frac{1}{1+(\sqrt{x})^2}\cdot\frac{1}{2\cdot\sqrt{x}}\,dx = 2\cdot\operatorname{arctg}\sqrt{x} + c$$

2. $$\int \frac{\ln^2 \ln x}{x\cdot\ln x}\,dx = \int \ln^2 \ln x\cdot\frac{1}{\ln x}\cdot\frac{1}{x}\,dx = \tfrac{1}{3}\cdot\ln^3 \ln x + c$$

3. $$\int \frac{x\cdot\arcsin^2 x^2}{\sqrt{1-x^4}}\,dx = \tfrac{1}{2}\cdot\int \arcsin^2 x^2\cdot\frac{1}{\sqrt{1-(x^2)^2}}\cdot 2x\,dx = \tfrac{1}{2}\cdot\tfrac{1}{3}\cdot\arcsin^3 x^2 + c$$

4. $$\int \underbrace{(1-2x)}_{u}\cdot\underbrace{3^{x+1}}_{\substack{v' \\ v=\frac{1}{\ln 3}\cdot 3^{x+1}}}\,dx = (1-2x)\cdot\tfrac{1}{\ln 3}\cdot 3^{x+1} - \int (-2\cdot\tfrac{1}{\ln 3}\cdot 3^{x+1}\,dx = (1-2x)\cdot\tfrac{1}{\ln 3}\cdot 3^{x+1} + \tfrac{2}{\ln^2 3}\cdot 3^{x+1} + c$$

5. $$\int \sqrt{x}\cdot\ln(1+\sqrt{x^3})\,dx \underbrace{=}_{\substack{t=1+\sqrt{x^3} \\ dt=\frac{3}{2}\cdot\sqrt{x}\,dx}} \tfrac{2}{3}\cdot\int \underbrace{1}_{\substack{u' \\ u=t}}\cdot\underbrace{\ln t}_{v}\,dt = \tfrac{2}{3}\cdot\left(t\cdot\ln t - \int t\cdot\frac{1}{t}\,dt\right) =$$

$$= \tfrac{2}{3}\cdot(t\cdot\ln t - t) + c = \tfrac{2}{3}\cdot\left((1+\sqrt{x^3})\cdot\ln(1+\sqrt{x^3}) - (1+\sqrt{x^3})\right) + c$$

6. $$\int \frac{4x+8}{x^2+2x+17}\,dx = 2\cdot\int \frac{2x+2}{x^2+2x+17}\,dx + \int \frac{4}{(x+1)^2+16}\,dx =$$

$$= 2\cdot\ln\left|x^2+2x+17\right| + \int \frac{\frac{1}{4}}{(\frac{x+1}{4})^2+1}\,dx = 2\cdot\ln(x^2+2x+17) + \operatorname{arc\,tg}(\tfrac{x+1}{4}) + c$$

7. $$\frac{8-x}{x^2-x-6} = \frac{A}{x+2}+\frac{B}{x-3} = \frac{A\cdot(x-3)+B\cdot(x+2)}{x^2-x-6} \Rightarrow \begin{matrix} A+B=-1 \\ -3A+2B=8 \end{matrix} \Rightarrow A=-2,\ B=1, \quad \text{s így}$$

$$\int \frac{8-x}{x^2-x-6}\,dx = \int \frac{-2}{x+2}\,dx + \int \frac{1}{x-3}\,dx = -2\cdot\ln|x+2| + \ln|x-3| + c$$

8. $$\int \frac{1}{\cos x+2\sin x+3}\,dx \underbrace{=}_{\substack{t=\operatorname{tg}\frac{x}{2},\ \cos x=\frac{1-t^2}{1+t^2} \\ \sin x=\frac{2t}{1+t^2},\ dx=\frac{2}{1+t^2}\,dt}} \int \frac{1}{\frac{1-t^2}{1+t^2}+\frac{4t}{1+t^2}+3}\cdot\frac{2}{1+t^2}\,dt = \int \frac{1}{1+(1+t)^2}\,dt = \operatorname{arc\,tg}(1+\operatorname{tg}\tfrac{x}{2}) + c$$

9. $$\int_1^2 \frac{x-1}{\sqrt{x^2-2x+4}}\,dx = \int_1^2 \frac{2x-2}{2\cdot\sqrt{x^2-2x+4}}\,dx = \left[\sqrt{x^2-2x+4}\right]_{x=1}^{x=2} = 2-\sqrt{3}$$

10. $$\int_0^{\pi/2} \cos^5 x\cdot\sin^4 x\,dx = \int_0^{\pi/2} \cos x\cdot(1-\sin^2 x)^2\cdot\sin^4 x\,dx = \left[\frac{\sin^5 x}{5}-\frac{2\cdot\sin^7}{7}+\frac{\sin^9}{9}\right]_{x=0}^{x=\pi/2} = \tfrac{1}{5}-\tfrac{2}{7}+\tfrac{1}{9}$$