II. ZÁRTHELYI MEGOLDÁSOK

2003. december 15.

B.

1. $$\int \frac{\sin x}{3+\cos^2 x}\,dx = -\frac{1}{\sqrt{3}}\cdot\int \frac{\frac{1}{\sqrt{3}}\cdot\sin x}{1+\left(\frac{1}{\sqrt{3}}\cdot\cos x\right)^2}\,dx = -\frac{1}{\sqrt{3}}\cdot\operatorname{arctg}\left(\frac{1}{\sqrt{3}}\cdot\cos x\right)+c$$

2. $$\int \frac{x+1}{\sqrt{3-2x-x^2}}\,dx = -\int \frac{-2-2x}{2\cdot\sqrt{3-2x-x^2}}\,dx = -\sqrt{3-2x-x^2}+c$$

3. $$\int (12x^5+3\sin 2x)\cdot(2x^6+3\sin^2 x+12)^5\,dx = \frac{1}{6}\cdot(2x^6+3\sin^2 x+12)^6+c$$

4. $$\int 3^{2x+1}\cdot\sqrt{9^x+9}\,dx = \frac{3}{\ln 9}\cdot\int 9^x\cdot\ln 9\cdot\sqrt{9^x+9}\,dx = \frac{3}{2\cdot\ln 3}\cdot\frac{2}{3}\cdot\sqrt{(9^x+9)^3}+c$$

5. $$\int \underbrace{(x-1)}_{u}\cdot\underbrace{\sin 3x}_{v'}\,dx = (x-1)\cdot\left(-\tfrac{1}{3}\cdot\cos 3x\right) - \int\left(-\tfrac{1}{3}\cdot\cos 3x\right)dx = (1-x)\cdot\left(\tfrac{1}{3}\cdot\cos 3x\right)+\tfrac{1}{9}\cdot\sin 3x+c$$
   $v=-\frac{1}{3}\cdot\cos 3x$

6. $$\int \sqrt{x}\cdot\operatorname{arctg}\sqrt{x^3}\,dx \underbrace{=}_{\substack{t=\sqrt{x^3}\\ dt=\frac{3}{2}\cdot\sqrt{x}\,dx}} \frac{2}{3}\cdot\int \underbrace{1}_{u'}\cdot\underbrace{\operatorname{arctg} t}_{v}\,dt = \frac{2}{3}\cdot\left(t\cdot\operatorname{arctg} t - \int\frac{t}{1+t^2}\,dt\right) =$$
   $u=t$

   $$= \frac{2}{3}\cdot\left(t\cdot\operatorname{arctg} t - \frac{1}{2}\cdot\ln(1+t^2)\right)+c = \frac{2}{3}\cdot\sqrt{x^3}\cdot\operatorname{arctg}\sqrt{x^3} - \frac{1}{3}\cdot\ln(1+x^3)+c \qquad (x>0)$$

7. $$\int \frac{4x-2}{x^2-2x+5}\,dx = 2\cdot\int\frac{2x-2}{x^2-2x+5}\,dx + \int\frac{2}{(x-1)^2+4}\,dx =$$

   $$= 2\cdot\ln\left|x^2-2x+5\right| + \int\frac{\frac{1}{2}}{\left(\frac{x-1}{2}\right)^2+1}\,dx = 2\cdot\ln\left|x^2-2x+5\right| + \operatorname{arctg}\left(\frac{x-1}{2}\right)+c$$

8. $$\frac{x+14}{x^2+3x-4} = \frac{A}{x-1}+\frac{B}{x+4} = \frac{A\cdot(x+4)+B\cdot(x-1)}{x^2+3x-4} \Rightarrow \begin{matrix} A+B=1\\ 4A-B=14\end{matrix} \Rightarrow A=3,\ B=-2,$$ s így

   $$\int\frac{x+14}{x^2+3x-4}\,dx = \int\frac{3}{x-1}\,dx - \int\frac{2}{x+4}\,dx = 3\cdot\ln|x-1| - 2\cdot\ln|x+4|+c$$

9. $$\int\cos^3(x-1)\cdot\sin^4(x-1)\,dx = \int\cos(x-1)\cdot(1-\sin^2(x-1))\cdot\sin^4(x-1)\,dx = \frac{\sin^5(x-1)}{5}-\frac{\sin^7(x-1)}{7}+c$$

10. $$\int\frac{2}{3\cos x+5}\,dx \underbrace{=}_{\substack{t=\operatorname{tg}\frac{x}{2},\ \cos x=\frac{1-t^2}{1+t^2}\\ dx=\frac{2}{1+t^2}\,dt}} \int\frac{2}{3\cdot\frac{1-t^2}{1+t^2}+5}\cdot\frac{2}{1+t^2}\,dt = \int\frac{\frac{1}{2}}{1+\left(\frac{1}{2}t\right)^2}\,dt = \operatorname{arctg}\left(\frac{1}{2}\cdot\operatorname{tg}\frac{x}{2}\right)+c$$